



## EPS/METOP

### Technical Note on Orbit Prediction

#### *EPSFDS*

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## DOCUMENT STATUS SHEET

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1.0	08/08/97	46	Minor corrections	
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## 1. INTRODUCTION

### 1.1 PURPOSE

The purpose of this document is to present the results from the study on EPS/METOP Orbit Prediction Techniques. The main parts of this study are:

1. **On-Ground Orbit Prediction Accuracy:** This section analyses the accuracy that can be obtained during the orbit prediction, for intervals of time up to 1 month. Real data from ERS-2 are used to determine the effect of some variables such as model complexity, OD length and solar activity. A simulation is performed for METOP for an initial estimation of the prediction accuracy that can be achieved.

2. **Analytical Models for Local Users:** This section studies several analytical models that can be used in order to estimate the position of METOP at a given time, using a simple set of parameters. The accuracy of the estimation is computed and an optimal model is recommended. The end-to-end performances of the OD/OP process are analysed.

Some conclusions and recommendations are presented at the end of this document.

### 1.2 SCOPE

This technical note is GMV's second delivery in the frame of the contract with EUMETSAT "EPS/METOP FLIGHT DYNAMIC SYSTEM STUDY". It documents the results from the following work package:

- **WP-2300: Orbit Prediction:** on the basis of the results of previous WP's 2100 and 2200 it shall be assessed the end-to-end performances of the orbit determination-orbit prediction tasks for varying extrapolation intervals (from 12 h up to 1 month). A thorough trade-off between algorithm complexity and prediction accuracy shall be performed; the influence of external phenomena such as solar activity shall be clearly analysed. As a result from the analyses a recommended orbit model shall be proposed taking into account the users requirements. As in the preceding WP's the performances that can be expected with such a model shall be assessed by means of suitable computer simulations, and recommendations on system configuration shall be derived.

It is scheduled to present these results in a meeting with EUMETSAT on August 8th, 1997. A final report will be delivered at the end of the project in which possible extensions to this document will be included, as well as the other technical notes that will be delivered for the rest of the work packages. Figure 1-1 shows the structure of the study, where the completed work packages have been checked out.

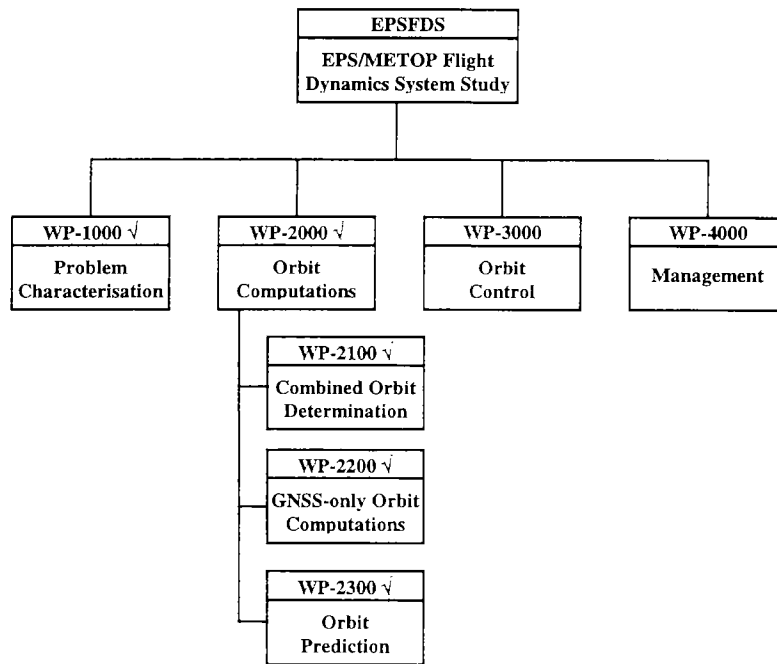


Figure 1-1: Work Package Structure

### 1.3 DEFINITIONS AND ACRONYMS

<b>AOCS</b>	Attitude and Orbit Control System
<b>AR</b>	Auto Regressing
<b>AS</b>	Anti Spoofing
<b>C/A</b>	Coarse Acquisition
<b>c.p.r.</b>	Cycle Per Revolution
<b>CPU</b>	Central Processing Unit
<b>DOP</b>	Dilution of Precision
<b>ECRV</b>	Exponentially Correlated Random Variable
<b>EPS</b>	EUMETSAT Polar System
<b>ERS</b>	European Remote-sensing Satellite
<b>ESA</b>	European Space Agency
<b>ESOC</b>	European Space Operations Centre
<b>ESTEC</b>	European Space Research and Technology Centre
<b>EURECA</b>	European REtrieval CArrier
<b>FD</b>	Failure Detection
<b>FDS</b>	Flight Dynamics System
<b>FFT</b>	Fast Fourier Transform
<b>FI</b>	Failure Identification
<b>FOV</b>	Field of View
<b>GEM</b>	Gravity Earth Model
<b>GLONASS</b>	Global Orbiting Navigation Satellite System
<b>GNSS</b>	Global Navigation Satellite System
<b>GPS</b>	Global Positioning System
<b>GRAS</b>	GNSS Receiver for Atmospheric Sounding
<b>HDOP</b>	Horizontal Dilution of Precision
<b>ISN</b>	Institute of Satellite Navigation (University of Leeds)
<b>JGM</b>	Joint Gravity Model
<b>KF</b>	Kalman Filter
<b>LEO</b>	Low Earth Orbit

<b>MEO</b>	Medium Earth Orbit
<b>METOP</b>	Meteorological Operational Satellite
<b>MMCC</b>	Mission Management and Control Centre
<b>MPTS</b>	Multi-Purpose Tracking System
<b>OD</b>	Orbit Determination
<b>OP</b>	Orbit Prediction
<b>P</b>	Precise code
<b>PDOP</b>	Position Dilution of Precision
<b>POD</b>	Precise Orbit Determination
<b>PRARE</b>	Precise RAnge and Range-ratE
<b>RAIM</b>	Receiver Autonomous Integrity Monitoring
<b>RMS</b>	Root mean square
<b>RSS</b>	Root Sum Square
<b>RTF</b>	Rationale Transfer Function
<b>SA</b>	Selective Availability
<b>SOAP</b>	Software Tool for Orbit and Attitude Determination Algorithms Performance Analysis
<b>SLR</b>	Satellite Laser Ranging
<b>UTC</b>	Universal Time Co-ordinated
<b>VDOP</b>	Vertical Dilution of Precision
<b>Wrt</b>	with respect to
<b>WP</b>	Work Package
<b>WWW</b>	World Wide Web



## 2. REFERENCES

### 2.1 APPLICABLE DOCUMENTS

N/A

### 2.2 REFERENCE DOCUMENTS

Reference	Title	Code	Issue	Date
[DS.1.]	Manual de la calidad	GMV-SGC-MAN-001	1.0	27/5/96
[DS.2.]	Glosario de términos y acrónimos	GMV-SGC-REF-001	1.0	29/5/96
[DS.3.]	Metodología de desarrollo	GMV-SGC-REF-003	1.0	30/5/96
[DS.4.]	Procedimiento de elaboración de documentos	GMV-SGC-PRO-001	1.0	31/5/96
[DS.5.]	Procedimiento de elaboración de planes de la calidad	GMV-SGC-PRO-020	1.0	18/7/96
[DS.6.]	Duque, P. (1987): Aerodynamic Forces and Moments of Free Molecular Flow	ESOC/OAD WP 347	N/A	1987
[DS.7.]	METOP Phase B Study. Orbit and Payload Coverage Analysis Report	MO-RP-MMB-SY-0047	1.0	10/1996
[DS.8.]	GMV, Study of Orbit and Attitude Determination Techniques for Low-Earth Observations Systems (ATLEOS)	GMV/SA 2116/95	1.0	11/1995
[DS.9.]	ESA, ESTEC, EUMETSAT, EPS/METOP System Requirement Document	EPS/SYS/REQ/93001	2.0	6/1996
[DS.10.]	Casotto, S. and Dow, J. M.; Orbit Determination of Low-Earth Satellites via the Global Positioning System (GPS)	ESA/ESOC OAD Working Paper No. 479	1.0	4/1993

[DS. 11.]	Potti, J., Peláez, A., "SOAP S/W User's Requirements Document", Study on Orbit Determination for Satellites at HEO (ODISHEO).	GMVSA 2079/94	1.0	8/1994
[DS. 12.]	Potti, J., Peláez, A., "SOAP S/W Architectural Design Document", GMV, Study on Orbit Determination for Satellites at HEO (ODISHEO).	GMVSA 2090/94	1.0	8/1994
[DS. 13.]	EUMETSAT, "EPS/METOP Flight Dynamics System Study"	ITT No. 96/156		
[DS. 14.]	GPS NAVSTAR. Global Positioning System, Standard Positioning Service, Signal Specification.	Navtech Seminars		10/1995
[DS. 15.]	T. A. Morley (ESA), "A SPOT Orbit Model on board ARTEMIS and SPOT-4"	OAD Working Paper No. 444		7/1991
[DS. 16.]	T.D.G. Clark et al., "Study on Forecasting of Solar and Geomagnetic Activity", ESA Study	British Geological Survey Technical Report WM/94/22C		1994
[DS. 17.]	M. M. Romay-Merino, G. García-Julián, "EPS/METOP Technical Note on Orbit Determination"	GMVSA 2108/97	2.0	8/1997
[DS. 18.]	MATRA-MARCONI SPACE, "METOP PHASE B STUDY, Orbit and Payload Coverage Analysis Report"	MO-RP-MMB-SY-0047	1.0	10/1996
[DS. 19.]	Mats Rosengren "The Orbit Control of ERS-1"	AAS 93-308		
[DS. 20.]	Mats Rosengren "Improved Technique for Passive Eccentricity Control"	AAS 89-155		
[DS. 21.]	Mats Rosengren "ERS Orbit Control"	Proceedings of the ESA Symposium on Spacecraft Flight Dynamics. Darmstadt, 30/9 - 4/10 1991		10/1991
[DS. 22.]	H. Klinkrad (ESA) "Semi-Analytical Theory for Precise Single Orbit Predictions of ERS-1"	ER_RP-ESA-SY-004	1.0	6/1987

Table 2-1: List of Reference Documents

### **3. ON-GROUND ORBIT PREDICTION ACCURACY**

#### **3.1 INTRODUCTION**

The orbit prediction problem involves many variables, some of which are very difficult to predict accurately. The solar activity and local air density evolve in a way that is hard to know in advance. A simulation of the circumstances that will be found during the real process of the orbit prediction should take into account these variables. This is an extremely complicated task.

For the purpose of this study, a thorough sensitivity study using real ERS-2 data has been completed. In this analysis the most relevant parameters affecting the orbit prediction accuracy have been analysed:

- Drag coefficient estimation method
- Gravity field model
- OD arc length
- Use of state-of-the-art models
- Solar activity

The following section will show the conclusions, which can be easily extrapolated to METOP. They will be applied to an attempt to simulate the process of orbit prediction for METOP. Some recommendations regarding the OP method that should be used for METOP are presented at the end.

### 3.2 ERS-2 ORBIT PREDICTION

The influence of factors such as the OD arc length,  $C_D$  estimation method (in particular, the frequency of the estimation), dynamic model and solar activity, on the orbit prediction accuracy is very difficult to simulate in a realistic way. Therefore, real ERS-2 data have been used to perform a study of these factors. The conclusions drawn from it can be extrapolated to METOP since it has already been proven in ref. [DS.18.] that the two satellites have a very similar behaviour and the sensitivity of their orbits to most variables is analogous. In fact, using ERS-2 conclusions for METOP should be a **conservative approach** due to the fact that METOP is less sensitive to drag (which will be the major source of error during OP) due to its higher orbit.

Considering the manoeuvres, shown on Table 3-1, a long period without manoeuvres was chosen. The period considered started on March 20, 1997 at noon and lasted 30 days. A longer period cannot be used due to the manoeuvres. It is important to notice that the period of time considered corresponds to a relatively low solar activity interval. The evolution of the prediction error through times of low and high solar activity will be shown later in this section.

Manoeuvre start		Calibrated $\Delta V$ (m/s)			Type	Error
Date	Time	Rad	Along	Cross		%
1/07/97	1:12:22 AM	0	0.006	0	INP FCM	-8.25
30/05/97	1:52:18 AM	0	0.007	0	INP FCM	-7.61
29/04/97	1:43:24 AM	0	0.002	0	INP FCM	-8.69
24/04/97	4:38:50 AM	0.001	0.017	-0.001	INP FCM	-4.09
24/04/97	3:48:36 AM	0.001	0.022	0.002	INP FCM	-3.09
22/04/97	7:07:08 PM	0.003	-0.069	0.006	INP FCM	-3.46
<del>22/04/97</del>	<del>5:41:03 AM</del>	<del>0.043</del>	<del>0.043</del>	<del>-1.098</del>	<del>OOP OCM</del>	<del>0.31</del>
<del>20/03/97</del>	<del>12:58:49 AM</del>	<del>0</del>	<del>0.007</del>	<del>0</del>	<del>INP FCM</del>	<del>-7.50</del>
15/02/97	11:16:27 PM	0	-0.01	0	INP FCM	-6.33
13/02/97	6:18:10 AM	0	-0.004	0	INP FCM	-7.88
10/02/97	6:47:23 PM	0	0.009	0.001	INP FCM	-2.77
10/02/97	5:56:59 PM	0.002	0.054	-0.003	INP FCM	-1.13
10/02/97	6:12:33 AM	0.045	-0.047	-1.232	OOP OCM	0.06
28/01/97	12:48:50 AM	0	0.001	0	INP FCM	-9.64
7/01/97	1:58:35 AM	0	0.004	0	INP FCM	-9.2
20/12/96	1:28:30 AM	0	0.002	0	INP FCM	-13.02
11/12/96	3:18:39 AM	0	-0.003	0	INP FCM	-3.21
3/12/96	8:03:11 PM	0	0.005	0	INP FCM	-7.91
3/12/96	7:12:45 PM	0.003	0.08	-0.004	INP FCM	-0.06
3/12/96	5:40:31 AM	0.062	-0.064	-1.561	OOP OCM	1.37

Table 3-1: ERS-2 Manoeuvres

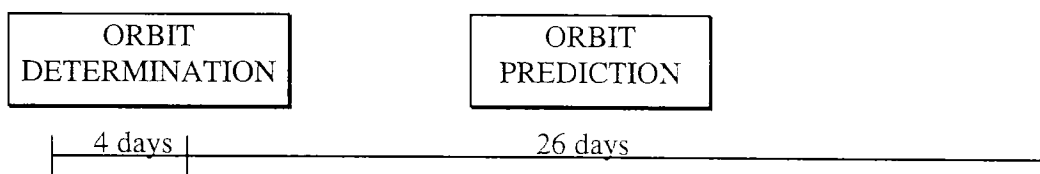
NOTE: INP FCM = In-Plane, Fine Control Mode; OOP OCM = Out-of-plane, Orbit Control Mode

Precise ERS-2 orbits (accurate to cm in the radial component and below 1 meter in the other two) have been retrieved for the period of time defined in the previous paragraph. They will be used as a reference for the comparisons with the orbits propagated with different methods during this study.

Real ERS-2 measurements will be used for the process of OD/OP. It was noticed that the Doppler measurements from Kiruna during the period of time between the 15<sup>th</sup> of February and the 20<sup>th</sup> of March (a longer interval) considered were unacceptable due to an anomaly and had been rejected during the operational orbit determination. That is the reason why the interval starting on March 20 was used instead.

For the purpose of this study, the **nominal conditions** included a 4-day orbit determination followed by a 26-day orbit prediction (this is limited by the period without manoeuvres). The OD was done with the model described on the following page, which is identical to the one suggested for METOP's OD excepting two points:

- The Earth gravity model is the **JGM-3 70x70** model instead of 36x36. This should improve the estimation accuracy of the  $C_D$  during the OD, which will allow a better orbit prediction. This effect was shown in section 4.3.1 of the Technical Note on Orbit Determination. The effect of using the 36x36 model instead of the 70x70 will be analysed during this study.
- The  $C_D$  is estimated as a constant for the whole OD period and used during the OP. When only OD is done it is advisable to use one  $C_D$  per day, but when it is followed by an OP the average should be used. Otherwise the error in the propagation will grow significantly. This fact will also be discussed during the study. This modification has been implemented as a result of some preliminary tests.



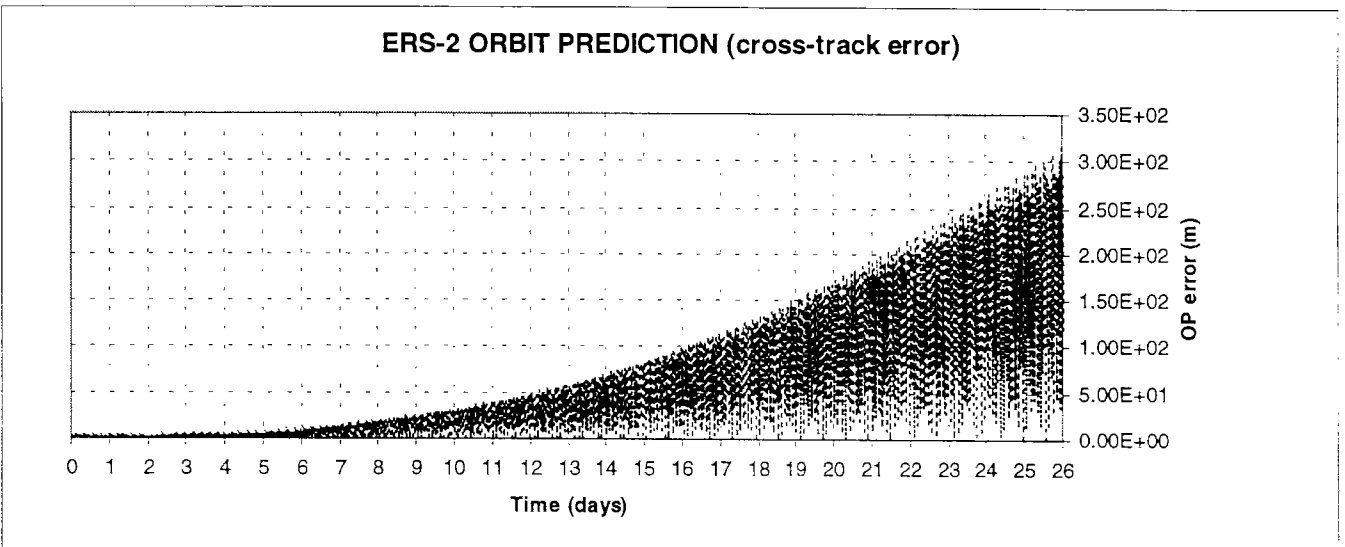
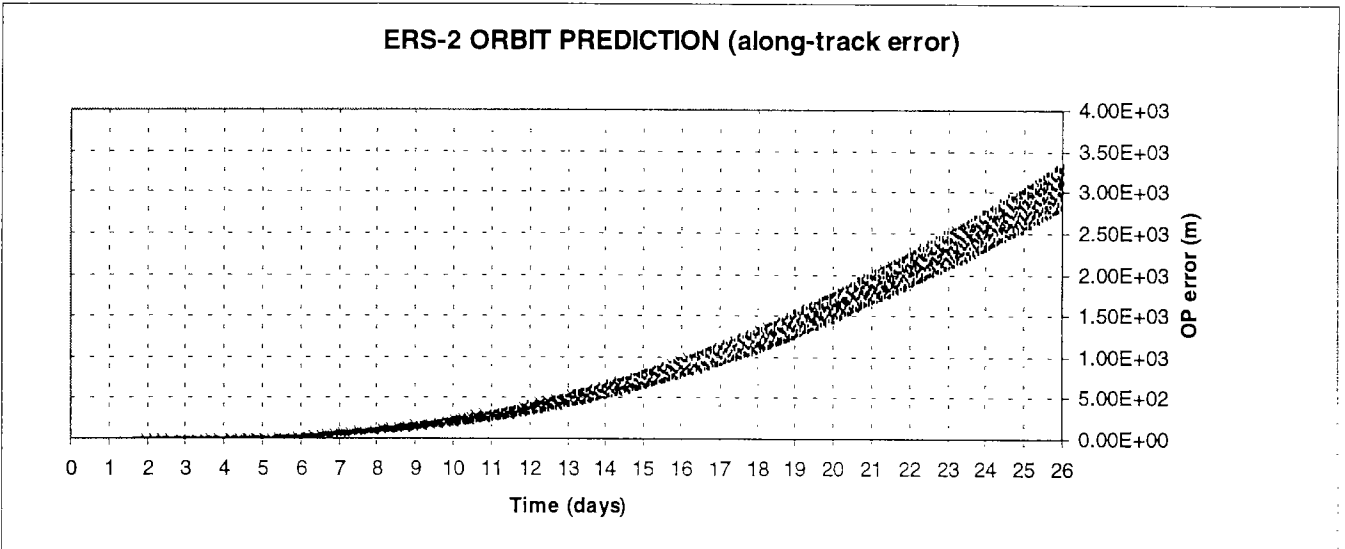
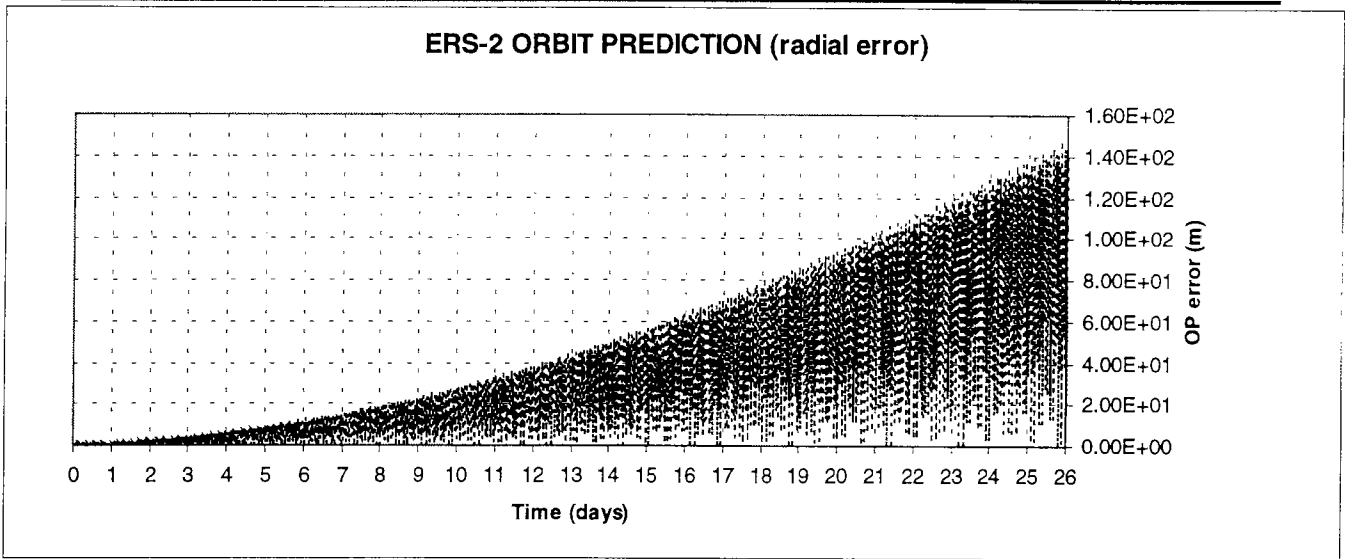
### **Dynamics:**

- JGM-3 (70x70) Earth gravity model
- MSIS density model, the latest available version is the MSIS-90 model, but the MSIS-83 is already providing accurate ERS orbits.
- Cannon-ball model for drag and solar radiation pressure. A single scale factor is estimated for the whole OD and used for the propagation (OP)
- Luni-solar gravity
- Ocean tide perturbations neglected
- Solid tide perturbations
- Albedo and infrared radiation perturbations neglected
- One cycle per revolution along-track and cross-track accelerations per arc

### **Measurements processing:**

- Hopfield tropospheric correction
- Rawer Bent ionospheric correction
- Spacecraft transponder delay and ground calibrations
- Centre of mass corrections do not need to be considered

Figure 3-1 shows the evolution of the resulting prediction error when this model is used.



*Figure 3-1: ERS-2 Orbit Prediction Error*

Table 3-2 shows the values of the prediction error at several points. The prediction error in the three components oscillates with an amplitude that grows with time around a growing value. The values shown correspond to the maximum around each time. As expected, the along-track error is by far the largest, due to the drag coefficient variation.

Prediction Time	Radial (m)	Along-track (m)	Cross-track (m)
0 hours (OD)	0.34	1.48	1.35
12 hours	0.69	2.98	1.60
24 hours	1.10	6.35	1.65
36 hours	1.70	9.01	1.96
3 days	4.10	12.6	2.68
7 days	14.8	91.1	14.2
15 days	57.0	841.	82.5
26 days	143.	3357.	314.

*Table 3-2: ERS-2 Prediction Error*

The values shown in this table could be used as a conservative reference for METOP. However, we will show that significant deviations on the prediction error are typical for different epochs.

In the next subsections the influence of several factors on the prediction accuracy will be analysed.



### 3.2.1 Influence of $C_D$ estimation method

For the nominal case, an average  $C_D$  was estimated during the OD arc and used for the OP arc. A different approach had been used for the regular OD in the previous work package, in which a  $C_D$  per day was estimated. If the propagation is done with the last value at the end of the OD, a value far from the average  $C_D$  might be used, causing a high deviation from the real orbit during the propagation.

This was confirmed by the results obtained when changing the  $C_D$  strategy from the nominal model, so that a  $C_D$  per day is calculated and the last value obtained during OD is used for OP. The error in along-track component after 26 days grew from about 3.3 km to more than 4.6 km. This effect could be much higher if the OD is done in an interval where peaks of  $C_D$  are present at the end. If the same method is used with a 36x36 Earth gravity model instead of the nominal 70x70 (while still solving for a daily  $C_D$ ), the along-track error after 26 days grows up to 28 km. This is due to the fact that the calculated  $C_D$  is less stable when using a more simple gravity model, so the deviation from the average is higher, a fact that we already mentioned in section 4.3.1 of the Technical Note on Orbit Determination.

Therefore, **the  $C_D$  used during the orbit prediction should be the average of the  $C_D$ 's computed during the orbit determination.** The prediction error may grow significantly otherwise. In any case, it is obvious that a considerable uncertainty will remain and drag will be the biggest source of the orbit prediction error, highly related to the variations in the solar activity, which will be explained later on. The average  $C_D$  will ensure that the effect of the peaks in the solar activity has a smaller impact on the orbit prediction error.

### 3.2.2 Influence of gravitational model

The nominal case used a JGM-3 70x70 Earth gravity model. It was stated that it would improve the accuracy of the orbit prediction, mainly because the higher accuracy of the gravity model would ensure a better estimation of the drag coefficient during the OD, which would increase the quality of the OP.

In order to verify that statement, a deviation from the nominal case was tested, in which the JGM-36x36 gravity model was used for both the OD and OP segments. Table 3-3 shows the prediction error with both methods after 26 days. The propagation error in the along-track and cross-track components is doubled by the combined effect of a worse gravitational model and a less accurate estimation of the average  $C_D$ . The radial component changes very little. A similar effect was seen after 36 hours of OP. This confirms our expectations.

Earth Gravity Model	Radial (m)	Along-track (m)	Cross-track (m)
JGM-3 70x70 (nominal)	143.	3357.	314.
JGM-3 36x36	147.	7710.	610.

Table 3-3: ERS-2 Prediction Error after 26 days with different gravity models

The conclusion is that **the computational cost of the use of the 70x70 model is justified if accurate orbit propagation is intended.**

### 3.2.3 Influence of OD length

The nominal case used an OD length of 4 days before the propagation. The length of the determination interval may have an impact on the accuracy of the orbit prediction. In particular, a good estimation of the average  $C_D$  is needed for a good prediction. If the OD interval is too short, peaks in the  $C_D$  will not be smoothed and an inaccurate value will be used for the prediction. On the other hand, if the OD interval is too long a price will be paid in computational time. The experience with previous satellites similar to METOP, like ERS-1 and ERS-2, indicates that 4 days is a good reference value for the OD length and should not be reduced.

In order to see the influence of the OD length, a new case was considered extended the OD interval one day, keeping the beginning of the OP interval on the same epoch as the reference case. The presence of a manoeuvre right before the beginning of the nominal OD period made it necessary to estimate it during this new case. Table 3-4 shows the results.

OD length	Radial (m)	Along-track (m)	Cross-track (m)
4 days (nominal)	143.	3357.	314.
5 days	13.	4432.	363

Table 3-4: ERS-2 Prediction Error after 26 days with different OD lengths

When doing a five-day OD, the prediction error increases in the along-track and cross-track components. This may be due partially to the estimation of the manoeuvre. There is a definite improvement in the radial component. In any case, the results seem to indicate that the computational effort needed to extend the OD length to 5 days is not justified by a general improvement in accuracy. Therefore, **the orbit determination length should be around 4 days** for a good orbit prediction.

### 3.2.4 Influence of other model variables

An easy way to proof that further improvements in the dynamic model are not advisable is showing that the most complex model available provides an accuracy similar to the one already obtained with the nominal model.

A **complex model** has been defined with the following improvements over the nominal model:

- Ocean tide perturbations included (extended Schwiderski model, 30 constituents (8x8) )
- Variable area model considered for atmospheric drag and direct solar radiation pressure

Further improvements in the model were considered unnecessary. Table 3-5 shows the prediction error obtained with both the nominal and complex model after 26 days.

Dynamic model	Radial (m)	Along-track (m)	Cross-track (m)
Nominal	143.	3357.	314.
Complex	39.	3138.	408.

*Table 3-5: ERS-2 Prediction Error after 26 days with different dynamic models*

The prediction error given by the complex model is slightly better in the along-track component, significantly better in the radial component and worse in the cross-track component. In any case, **the increase in complexity is not justified by a significant improvement of the prediction error**. Therefore, the nominal model is still the best choice.

### 3.2.5 Influence of solar activity

It has already been mentioned that the variation in the solar activity creates a big source of uncertainty. The peaks in the solar activity modify the effect of the drag and solar radiation pressure on the orbit. These short-term changes are hard to predict. Long-term changes like cyclical variations of the solar activity are easier to account for. Obviously, the error during times of high activity will grow since the short-term effects mentioned earlier will account for a higher percentage of the perturbations.

To illustrate these facts, the evolution of the orbit prediction error through time is shown for ERS-1 and ERS-2 on Figure 3-2. The data shown correspond to the operational prediction of both satellites and show the experience from real satellites whose orbits are very similar to METOP. The figures show that **the prediction error after 6 days may vary more than a 400%** between different epochs. The peaks are more significant for longer prediction intervals.

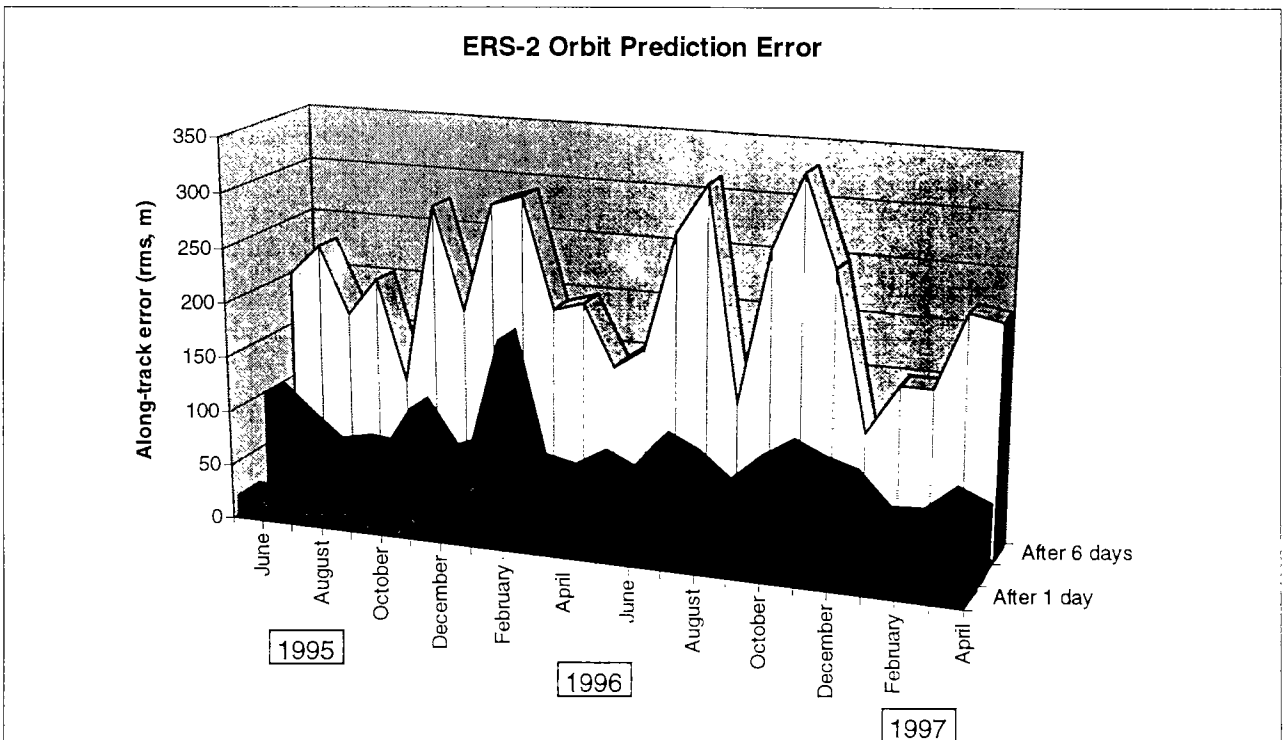
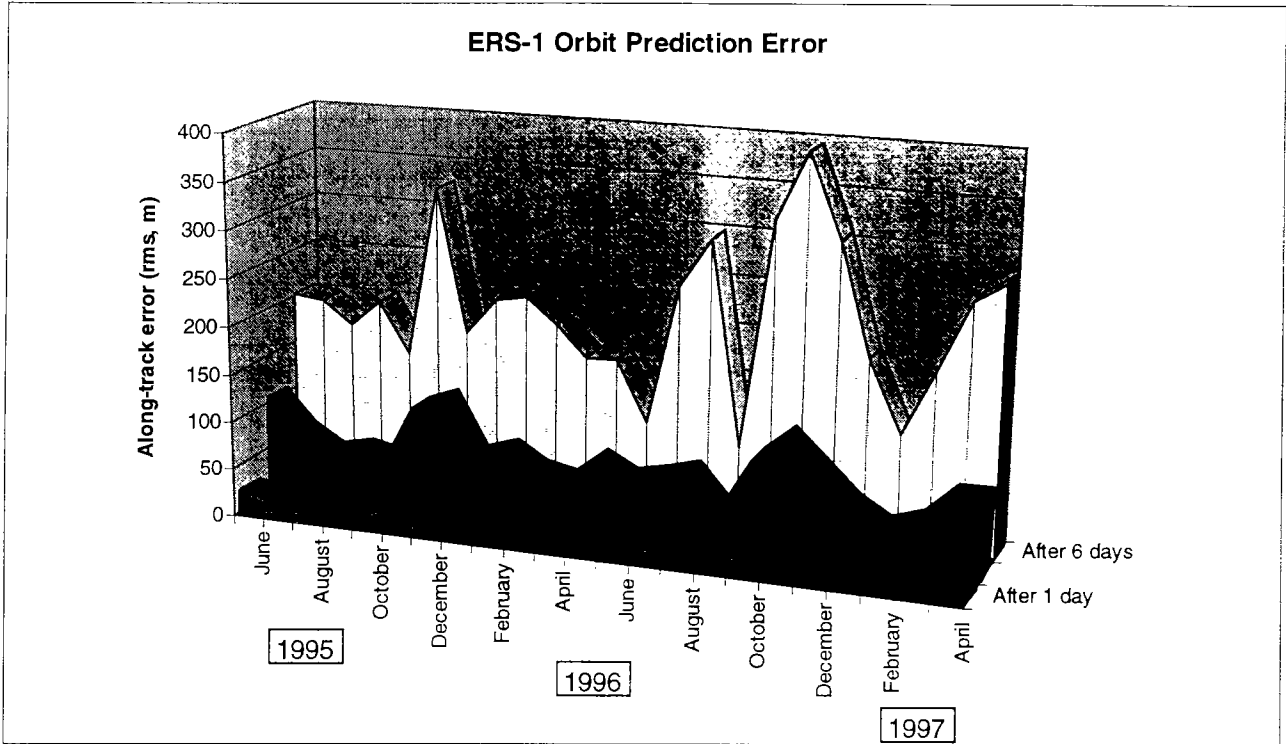


Figure 3-2: Orbit Prediction Error Evolution for ERS-1 and ERS-2

Figure 3-3 shows the evolution of the orbit prediction error for ERS-1 since it was launched in 1991. It can be compared with the evolution of the solar activity during the 90's, as measured by the solar activity flux index F10.7, shown on Figure 3-4.

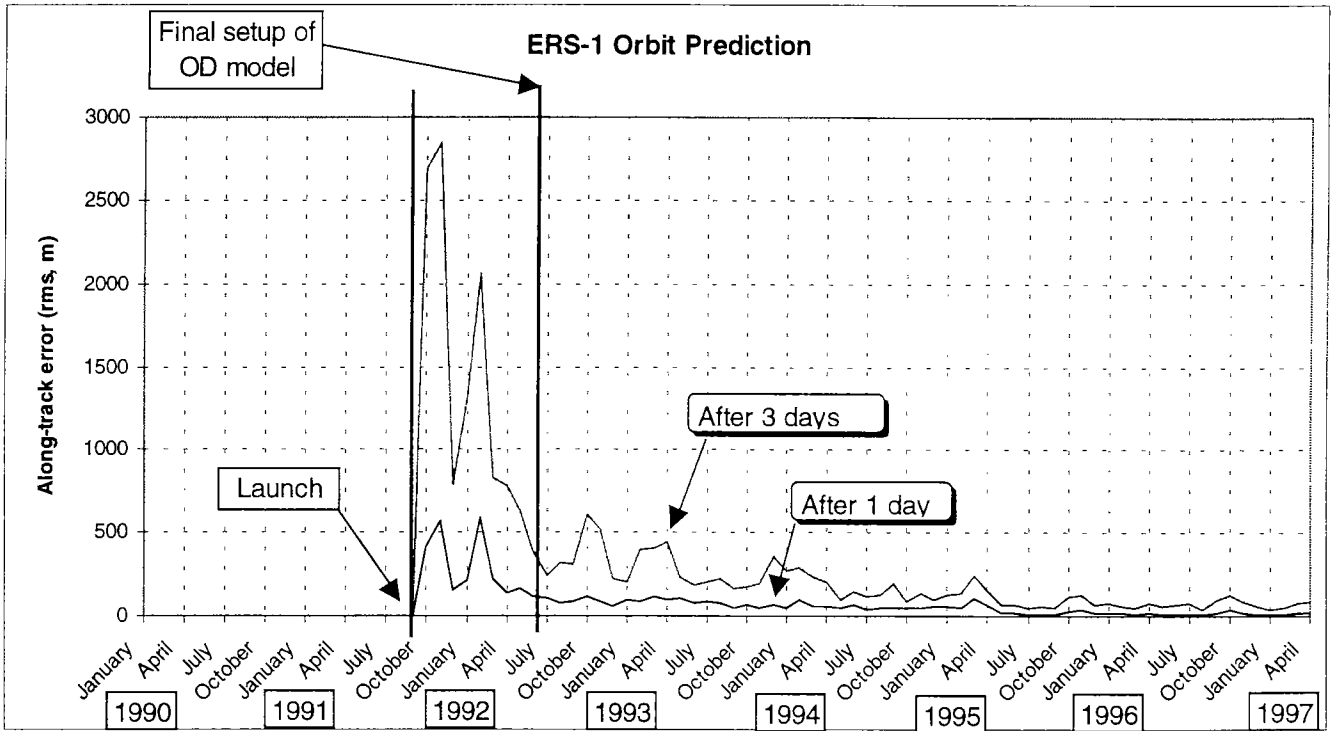


Figure 3-3: Orbit Prediction Error Evolution for ERS-1

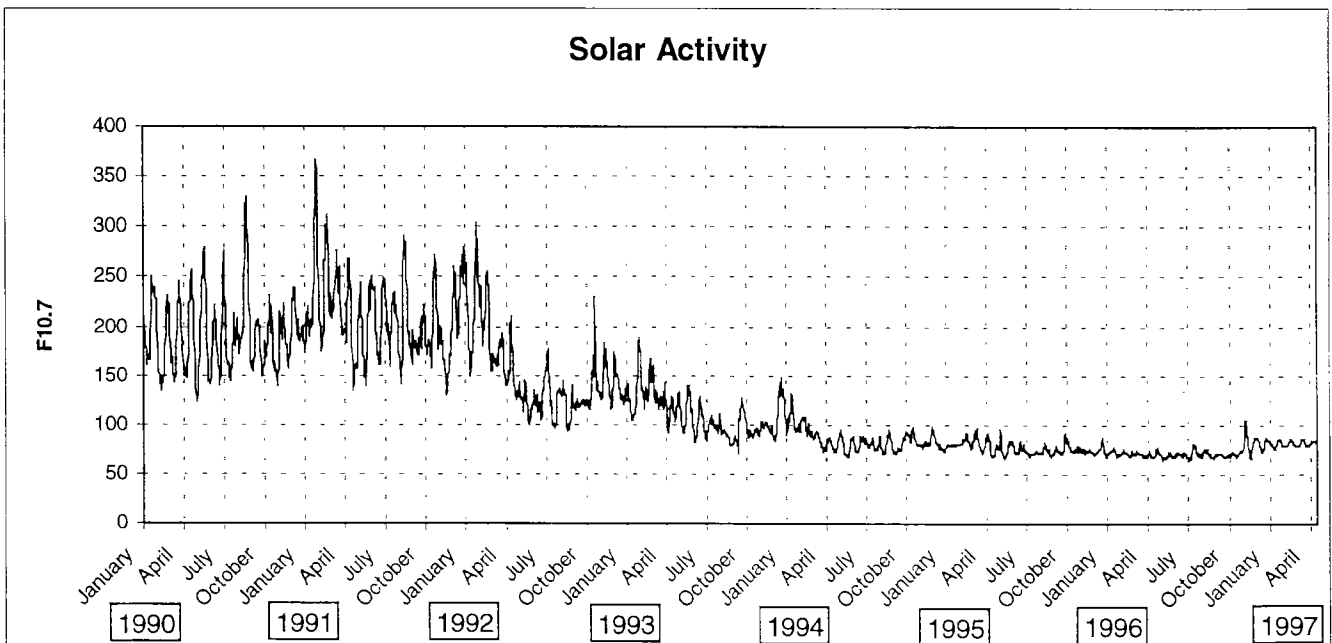


Figure 3-4: Solar Activity evolution in the 90's

The main point to be highlighted is the irregularity of the F10.7 data. There are long periods of low activity levels and shorter bursts of high activity levels, which appear to occur at random. The evolution of the prediction error shows some correlation with the changes in the solar activity. The peaks at the beginning of the life of ERS-1 correspond to the initial phases of the mission during which the tune-up of the models was done. It happened to occur during a high solar activity period, which increased the error. After the final set-up of the OD model was achieved, the prediction error in the along-track component after 3 days was taken under 1 km. **The peaks in solar activity cause an increase in the prediction error.**

Several attempts have been made trying to find a way to predict the solar activity, like the one described in [DS.16.]. They all fail to predict the larger geomagnetic storms, which, in general, do not show any periodic properties. In any case, even if the solar activity is predicted accurately it is not easy to correlate the evolution of the  $C_D$  with the solar activity. Thus, **a big uncertainty remains.**

It should be noticed that the values of the prediction error in the figures are higher than the ones obtained during the analysis of real ERS-2 data at the beginning of section 3.2. The main reason for this is the fact that the dynamic model used during the operational OP for ERS-1 and ERS-2 is simpler than the one used in this study. In particular, a 36x36 JGM-3 model is chosen for the gravitational model and the estimation of the  $C_D$  has not been optimised.

### 3.3 METOP ORBIT PREDICTION SIMULATION

The analysis of the Orbit Prediction problem for ERS-2 showed that the propagation error varies significantly depending on the time period considered. The unpredictable changes in the solar activity will modify the drag and solar radiation pressure forces acting on the satellite. The estimated  $C_D$  coefficient during the OD segment may be very different from the values of the  $C_D$  during the OP segment if the solar activity is changing and this will cause a large OP error. All these factors make the simulation of the OP process for METOP a very difficult task if a realistic approach is intended. **The results from ERS-2 provide a realistic view of the problem and its results can be extrapolated to METOP.**

However, a simulation has been done for METOP in order to see the influence of the error in the estimation of the  $C_D$  during the OD segment. Tracking data were simulated for METOP using the same model that was applied to simulate data for the study of orbit determination in the previous chapter, except for the fact that a single  $C_D$  was used for the whole interval. A full description of the conditions follows:

- **Epoch**

15th of March 1997 at 12:00

- **Reference frame:**

Mean equator and equinox of J2000.0 (i.e.)

- **Initial state**

Orbital parameter	METOP
Semi major axis (km)	7197.939472
Eccentricity	0.001165
Inclination (deg)	98.704663
Right ascension of the ascending node (deg)	136.61998
Argument of perigee (deg)	90.0000
True Anomaly (deg)	270.13359

- **Satellite characteristics**

	METOP
Satellite mass (kg)	4420.0
Satellite area for solar radiation pressure (m <sup>2</sup> )	36.00
Area to mass ratio (m <sup>2</sup> /kg)	0.008145

- **Force model**

The force model used is described in the table below:

Earth gravitational field	JGM-3 (70x70)
Direct solar perturbations	Included
Direct lunar perturbations	Included
Solid tide perturbations	Included
Ocean tide perturbations	Extended Schwiderski model, 30 constituents (8x8)
Atmospheric drag	Included, <b>Variable area model not considered</b>
Direct solar radiation pressure	Included, <b>Variable area model not considered</b>
Earth albedo perturbations	Neglected
Infrared radiation perturbations	Neglected
Empirical force parameters and orbital manoeuvres	Not considered



- **Arc length**

34 days

In order to improve the realism of the simulation, some **artificial errors** have been introduced in the model's parameters. These introduced errors are basically the same as the ones introduced for the ERS-2 case simulation that was done for the Orbit Determination, so they have already been described in the Technical Note on OD (ref. [DS.15.]). They include:

- Error (25 metres) introduced in the initial state vector for the orbit determination
- Random error introduced in the air drag (= 0.375) and solar radiation pressure (= 0.05) coefficients

The **model used for the determination and prediction** of the orbit is identical to the one used in the simulation that was done for the OD work package, except for the fact that during the OD a constant  $C_D$  is estimated and used for the OP:

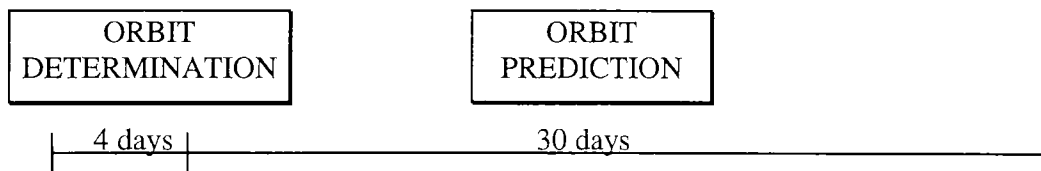
**Dynamics:**

- Truncated JGM-3 (36x36) Earth gravity model. This is used instead of the 70x70 model in order to include an error source in the gravitational field, which will make the simulation more realistic.
- MSIS density model
- Cannon-ball model for drag and solar radiation pressure. Constant  $C_D$  estimated.
- Luni-solar gravity
- Ocean tide perturbations neglected
- Solid tide perturbations
- Albedo and infrared radiation perturbations neglected
- One cycle per revolution along-track and cross-track accelerations per arc

**Measurements processing:**

- Hopfield tropospheric correction
- Rawer Bent ionospheric correction
- Spacecraft transponder delay and ground calibrations
- Centre of mass corrections do not need to be considered

The orbit determination segment is identical to the one studied in the OD work package and uses the same nominal model as the one used then, for a period of 4 days. An orbit prediction was performed with the same model, propagating the orbit for 30 days.



The analysis of the Orbit Prediction for ERS-2 showed that one of the major variables that determine the accuracy is the  $C_D$ . In order to analyse the influence of the error of the  $C_D$  estimated during the OD segment, a sensitivity study has been carried out. By looking at the value of the average  $C_D$  obtained during the corresponding segment generated in the simulated orbit, several cases with different error levels (for the drag coefficient used during the OD/OP process) have been considered. The table below shows the sensitivity of the prediction error after 26 days to the error of the  $C_D$  estimated during OD:

Error in $C_D$ %	Radial (m)	Along-track (m)	Cross-track (m)
0%	6.0	54.3	21.6
5%	5.6	502.	51.5
10%	5.2	953.	86.8
15%	4.9	1404.	121.
20%	4.6	1855.	157.
25%	4.4	2305.	193.

Table 3-6: METOP Prediction Error after 26 days with different levels of error in the  $C_D$



The error of the  $C_D$  estimated during OD has a big impact on the prediction error, as expected. The variation of the solar activity will cause a variation of the prediction error. It is not possible to provide a single number to determine the accuracy of the OP problem for METOP, since it will vary in a broad interval, probably similar to the one shown for ERS-1 and ERS-2 earlier.

Figure 3-5 shows the evolution of the predic

tion error for a 25% error in the  $C_D$  calculated during OD, which should not be used as an absolute reference.

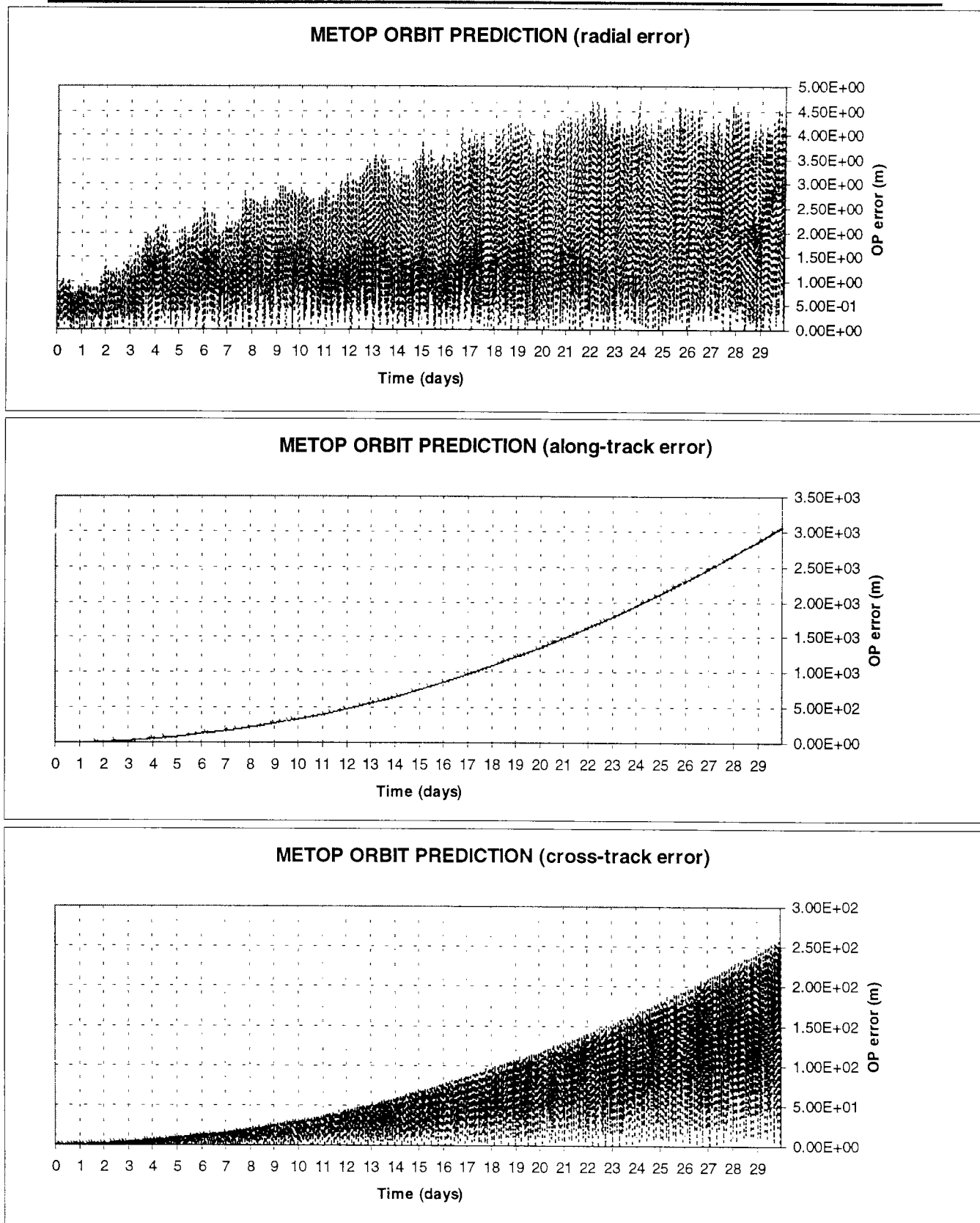


Figure 3-5: METOP Orbit Prediction Error for a 25% error in the  $C_D$

The table below shows the values of the prediction error at several points, for a 25% error in the  $C_D$  used for the propagation. The prediction error in the three components oscillates around a value that grows with time. The values shown correspond to the maximum around each time. Again, the real values may differ significantly from these values.

Prediction Time	Radial (m)	Along-track (m)	Cross-track (m)
0 hours (OD)	0.87	3.02	1.26
12 hours	1.00	5.21	1.53
24 hours	.85	4.21	2.20
36 hours	.85	7.33	2.35
3 days	1.39	39.7	5.46
7 days	2.21	179.	16.9
15 days	3.86	771.	68.6
26 days	4.38	2305.	193.
30 days	4.43	3070.	249.

*Table 3-7: METOP Prediction Error for a 25% error in the  $C_D$*

### 3.4 CONCLUSIONS FROM THE ON-GROUND ORBIT PREDICTION ANALYSIS

The following conclusions can be drawn from the analysis performed on the Orbit Prediction process:

- **The estimation of the  $C_D$  during the OD segment is critical** and will have a big impact on the accuracy of the OP. The model defined as “nominal” in section 3.2 is recommended. The following points are essential:
  - **The  $C_D$  used during the OP segment should be an average of the  $C_D$ 's estimated during the OD segment.** This is the best way to minimise the propagation error due to a bad estimation of the  $C_D$  for the OP segment.
  - **The JGM-3 70x70 Earth Gravity Model should be used for OD and OP.** It will ensure a more stable  $C_D$  during the OD segment and therefore a better prediction accuracy. It has been proven that the increase in complexity is justified by the improvement in the quality of the prediction.
  - **An OD segment of around 4 days should be considered prior to the OP.** This will ensure a good balance between parameters estimation and computational time.
  - **Further upgrades, such as the inclusions of ocean tide perturbations or variable area models are not recommended.** It has been proven that the improvement in accuracy obtained does not justify the complexity introduced in the model.
- **The variation of the solar activity introduces a big uncertainty in the resulting OP accuracy.** Variations of more than a 400% in the prediction error after 6 days are found in real ERS-1 and ERS-2 operations and should be expected for METOP. For this reason it is not possible to perform an accurate simulation of the OP process for METOP. The prediction error is expected to be comparable to the one obtained for the ERS satellites (along-track error after 6 days between 100 and 400 metres). However, a simulation has been done for METOP and different prediction error levels have been shown for different error levels in the  $C_D$  estimated during the OD segment, which is the major source of uncertainty. Table 3-8 summarises the prediction errors after several time lengths for METOP (simulation with a 25% error in the  $C_D$ ) and ERS-2 (real data).



Prediction Time	Radial (m)		Along-track (m)		Cross-track (m)	
	<i>METOP</i>	<i>ERS-2</i>	<i>METOP</i>	<i>ERS-2</i>	<i>METOP</i>	<i>ERS-2</i>
0 hours (OD)	0.87	0.34	3.02	1.48	1.26	1.35
12 hours	1.00	0.69	5.21	2.98	1.53	1.60
24 hours	.85	1.10	4.21	6.35	2.20	1.65
36 hours	.85	1.70	7.33	9.01	2.35	1.96
3 days	1.39	4.10	39.7	12.6	5.46	2.68
7 days	2.21	14.8	179.	91.1	16.9	14.2
15 days	3.86	57.0	771.	841.	68.6	82.5
26 days	4.38	143.	2305.	3357.	193.	314.
30 days	4.43	N/A	3070.	N/A	249.	N/A

*Table 3-8: METOP and ERS-2 Prediction Errors*

## 4. ANALYTICAL MODELS FOR LOCAL USERS

### 4.1 INTRODUCTION

Once the on-ground orbit determination and prediction is done, a set of parameters will be sent to the local users, who will estimate the orbit through a simple analytical model. There are many analytical models in the literature. For the purpose of this study, three models have been analysed:

- GPS model
- Extended GPS model
- SPOT model

An orbit model to be used by the local users is recommended in the light of the results from the analysis. This recommendation takes into account the users needs for **relatively high accuracy with a simplified orbit model** that can be easily specified and if possible procured or derived from commercially available products. An estimation of the performances that can be achieved with such a model is also shown.

**A software tool has been developed for the parameter estimation with the three models.** Figure 4-1 shows the process implemented. After reading the data from the orbit, an initial set of parameters is estimated from the first few points. With that initial set of parameters, the estimated position (according to the model) is calculated at each time  $t_k$  for which the orbit is given. The derivatives of the position with respect to each parameter are also calculated at each time  $t_k$ . The residuals (difference between the estimated and actual position components) and its rms are computed. A least-squares estimator gets a new set of parameters from the residuals and derivatives. This new set is fed back to the process for a new iteration. This algorithm is followed in order to determine the parameters that lead to the lowest position error.



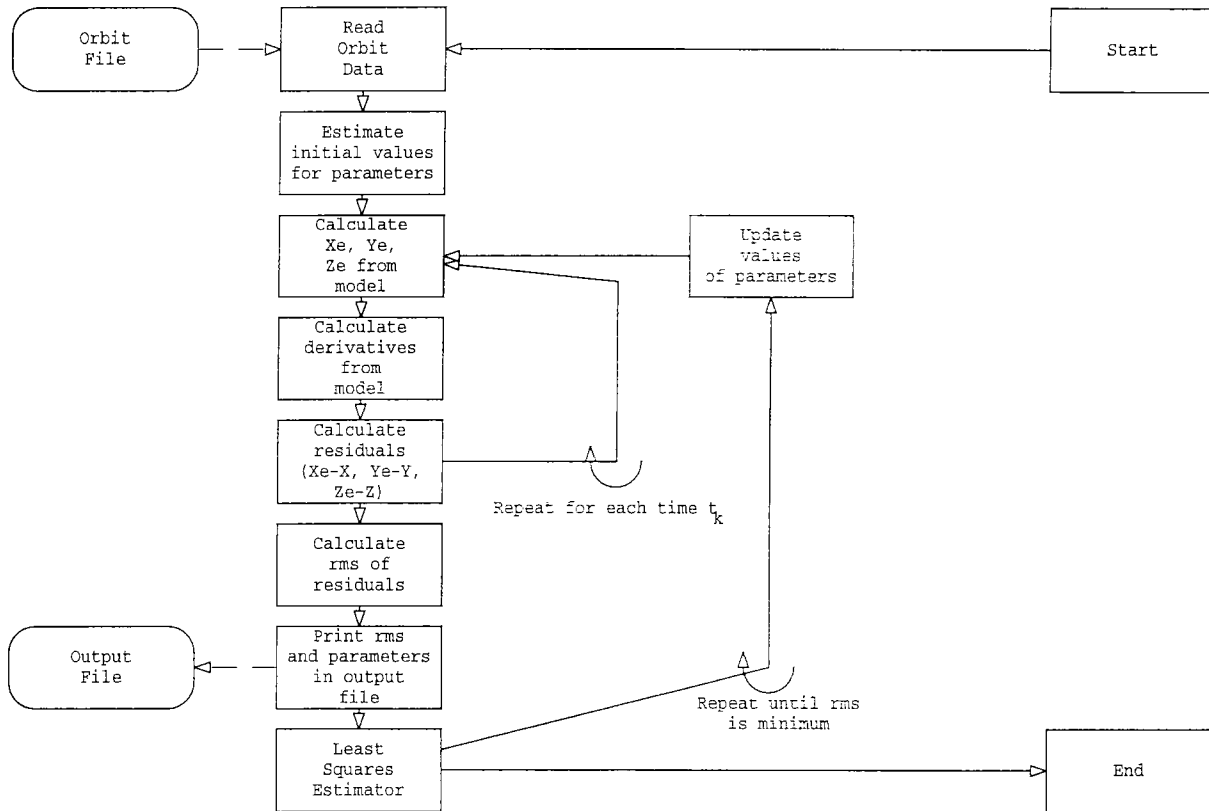


Figure 4-1: Orbit Model Parameter Estimation Algorithm

The calculation of the position for a given set of parameters and time usually implies the conversion from orbital elements to state vector (depending on the model). The derivatives are calculated by giving a small increment to each parameter and computing the increment in the position components.

**The speed of this process depends on the complexity of the model.** However, the determination of the position for a given set of parameters and epoch will be almost instantaneous at the local user. Therefore, **almost all the computational effort will take place before the parameters are sent to the user.**

## 4.2 DESCRIPTION OF THE MODELS

The different models analysed are briefly described in this section

### 4.2.1 GPS Model

The GPS Model corresponds to the one applied by the GPS receivers in order to calculate the position of the GPS satellites from the message sent by them. It is a very standard model and that is the reason why it has been deemed advisable for the purpose of this study. The availability of commercial products would make the implementation of the algorithm a simple process. The parameters included in the **GPS message** are defined below:

#### Time parameters

$t_{0e}$	Reference time for ephemeris parameters
$t_{0c}$	Reference time for clock parameters
$a_0, a_1, a_2$	Polynomial coefficients for clock correction (bias, drift and drift-rate)
IOD	Issue of data, arbitrary identification number

#### Keplerian parameters

$a^{1/2}$	Square root of the semi-major axis
$e$	eccentricity
$i_0$	inclination angle at reference time
$\Omega_0$	Right ascension of the ascending node at reference time
$\omega$	Argument of the perigee
$\bar{M}_0$	Mean anomaly at reference time

### **Perturbation parameters**

$\Delta n$	Mean motion difference from computed value
$\dot{\Omega}$	Rate of change of right ascension
$\dot{i}$	Rate of change of inclination
$C_{us}$	Amplitude of the sine harmonic correction to the argument of latitude
$C_{uc}$	Amplitude of the cosine harmonic correction to the argument of latitude
$C_{is}$	Amplitude of the sine harmonic correction to the angle of inclination
$C_{ic}$	Amplitude of the cosine harmonic correction to the angle of inclination
$C_{rs}$	Amplitude of the sine harmonic correction to the orbit radius
$C_{rc}$	Amplitude of the cosine harmonic correction to the orbit radius

The GPS model that we will consider is defined by the previous parameters except for the time parameters, which are not needed for our application, in which an accurate on-ground orbit determination is done and the clock errors are accounted for. Therefore, **the GPS model is defined by a set of 15 parameters** (Keplerian plus perturbation parameters).

The satellite position at a certain epoch can be determined from the previous set of parameters by calculating the perturbed Keplerian orbit that corresponds to the values of the parameters. The set will be valid for a certain period of time only. The calculation of the parameters is done by an optimisation in order to minimise the position error during that period.

The way the local user calculates the position of the satellite at a certain time  $t$  for a given period starting at  $t_{oe}$  and a set of parameters is explained below. There are two constants,  $GM = 3.9686005 \cdot 10^{14} \text{ m}^3/\text{s}^2$  (geocentric gravitational constant) and  $\omega_e = 7.2921151467 \cdot 10^{-5} \text{ rad/s}$  (earth rotation rate)

$t_k = t - t_{0e}$	Time elapsed since reference epoch $t_{0e}$	$u_k = \Phi_k + \delta u_k$	Corrected true argument of latitude
$a = (a^{1/2})^2$	Semi-major axis	$r_k = a(1 - e \cos E_k) + \delta r_k$	Corrected radius
$n_0 = \sqrt{\frac{GM}{a^3}}$	Computed mean-motion	$i_k = i_0 + \dot{i}t_k + \delta i_k$	Corrected inclination
$n = n_0 + \Delta n$	Corrected mean-motion	$X'_k = r_k \cos u_k$	X Position in the orbital plane
$M_k = M_0 + nt_k$	Mean anomaly	$Y'_k = r_k \sin u_k$	Y Position in the orbital plane
$E_k = M_k + e \sin E_k$	Eccentric anomaly (solved by iteration)	$\Omega_k = \Omega_0 + (\dot{\Omega} - \omega_e)t_k - \omega_e t_{0e}$	Corrected longitude of ascending node
$\cos v_k = \frac{\cos E_k - e}{1 - e \cos E_k}$	True anomaly	$X_k = X'_k \cos \Omega_k - Y'_k \sin \Omega_k \cos i_k$	Earth fixed geocentric satellite X coordinate
$\sin v_k = \frac{\sqrt{1 - e^2} \sin E_k}{1 - e \cos E_k}$	True anomaly	$Y_k = X'_k \sin \Omega_k + Y'_k \cos \Omega_k \cos i_k$	Earth fixed geocentric satellite Y coordinate
$\Phi_k = v_k + \omega$	True argument of latitude	$Z_k = Y'_k \sin i_k$	Earth fixed geocentric satellite Z coordinate
$\delta u_k = C_{uc} \cos 2\Phi_k + C_{us} \sin 2\Phi_k$	True argument of latitude correction		
$\delta r_k = C_{rc} \cos 2\Phi_k + C_{rs} \sin 2\Phi_k$	Radius correction		
$\delta i_k = C_{ic} \cos 2\Phi_k + C_{is} \sin 2\Phi_k$	Inclination correction		



#### 4.2.2 Extended GPS Model

The first results, which will be shown at the end of this chapter, indicated that the GPS model does not adapt well to a LEO satellite. Among other effects, the drag causes a decrease in the semi-major axis that is not important for a GPS satellite due to its high altitude, but it is significant for a LEO satellite.

An **extended GPS Model** has been defined in order to improve the estimation of the semi-major axis and its variation due to drag. The extended model is identical to the GPS model defined earlier except that the semi-major axis is expanded into  $a = a_0 + \dot{a}t_k$ , where  $\dot{a}$  is an additional parameter. Therefore, the extended GPS model requires **16 parameters**.

Although other extensions of the model could be considered, the results from the SPOT model will show that it is not worth it.

### 4.2.3 SPOT Model

The GPS Model works well with satellites in a medium or high orbit. For a satellite such as METOP, with a low orbit and therefore significant perturbations due to the non-sphericity of the earth and drag, a more accurate model may be required.

Reference [DS.15.] describes the orbit model that will be used for SPOT in the context of the ARTEMIS mission. SPOT is a set of LEO, sun-synchronous satellites with an orbit that is very similar to METOP's as shown on the table below:

<b>Orbital parameter</b>	<b>METOP (nominal)</b>	<b>SPOT (mean values)</b>
Semi major axis (km)	7197.939472	7200.6
Eccentricity	0.001165	0.001
Inclination (deg)	98.704663	98.7
Argument of perigee (deg)	90.00000	90.0

*Table 4-1: METOP vs. SPOT*

The generalised on-board orbit model given by ref. [DS.15.] was defined by performing a study of the major sources of perturbations acting on the satellite and obtaining an analytical approximation for them. All the equations are shown in the reference and will not be repeated here, except for the final expressions. The perturbations included in the model are:

- Secular and short period perturbations due to oblateness.  $J_2$
- Significant short-period perturbations due to  $J_{2,2}$
- Short-period position perturbations (due to gravity)
- Long-period perturbations (due to gravity)
- Atmospheric drag

When the analytical expressions for these perturbations are combined, the following set of equations (which define the model) is obtained:

$$\begin{aligned}
 t_k &= t - t_{oe} \\
 a_k &= P_1 \left( 1 + \frac{3}{2} P_9 \sin^2 P_4 \cos 2\bar{\alpha}_k \right) \\
 e_k \cdot \cos \omega_k &= P_2 - P_3 P_{13} t_k + P_9 \left[ \frac{7}{8} \sin^2 P_4 \cos 3\bar{\alpha}_k - \frac{3}{2} \left( \frac{5}{4} \sin^2 P_4 - 1 \right) \cos \bar{\alpha}_k \right] \\
 e_k \cdot \sin \omega_k &= P_3 + P_2 P_{13} t_k + P_9 \left[ \frac{7}{8} \sin^2 P_4 \sin 3\bar{\alpha}_k - \frac{3}{2} \left( \frac{5}{4} \sin^2 P_4 - 1 \right) \sin \bar{\alpha}_k \right] \\
 i_k &= P_4 + \frac{3}{8} P_9 \sin 2P_4 \cos 2\bar{\alpha}_k + \left[ \frac{P_{10}}{3 \sin P_4} \cos \left( \frac{4\pi t_k}{T_d} + P_{11} \right) \right] \\
 \Omega_k &= P_5 + P_7 t_k + \frac{3}{4} P_9 \cos P_4 \sin 2\bar{\alpha}_k \\
 \alpha_k &= \bar{\alpha}_k + \frac{3}{4} P_9 \left( \frac{5}{2} \sin^2 P_4 - 1 \right) \sin 2\bar{\alpha}_k + P_{10} \sin \left( \frac{4\pi t_k}{T_d} + P_{11} \right) \\
 \text{where} \\
 \bar{\alpha}_k &= P_6 + P_8 t_k + P_{12} t_k^2
 \end{aligned}$$

This model will provide the osculating orbital elements  $a_k$  (semi-major axis),  $e_k$  (eccentricity),  $i_k$  (inclination),  $\Omega_k$  (right ascension of ascending node),  $\omega_k$  (argument of perigee) and  $\alpha_k$  (argument of latitude,  $\alpha_k = \omega_k + M_k$ , where  $M_k$  is the mean anomaly) at each time  $t_k$  in seconds, for a given set of parameters  $P_1, \dots, P_{13}$ .  $T_d$  is 86400 seconds for a sun-synchronous satellite. This model is more complex than the GPS models so a small price has to be paid regarding computational time for both the on-ground parameter optimisation and user's position determination, although the latter is negligible. On the other hand, **only 13 parameters are needed to define the model.**

Reference [DS.15.] discusses some variations on this model. One of them involves the usage of 9 constants  $K_1, \dots, K_9$ , tailored to SPOT, plus 10 free parameters  $P_1, \dots, P_{10}$ . When this model was applied to METOP (estimating a new set of constants) it turned out to provide worse results when compared with the 13-parameter model described above, so it was disregarded.

### 4.3 ACCURACY OF THE MODELS

In order to estimate the position errors given by the orbit models described earlier, an orbit determination process was completed for METOP with the same models described in section 3.3, for a period of 36 hours. The satellite position was determined every 9 minutes (0.00625 days). **The resulting orbit was used to estimate the parameters that provide the lowest position error for each of the analytical models described earlier in this section.** This position error only includes the error of the analytical model when compared to the orbit calculated during the orbit determination. It does not include the error due to the OD process itself nor the errors caused by on-board clock inaccuracies (which will produce a mismatch between the reference time used for the determination of the parameters and the time used for the analytical estimation of the position at the local user).

The model fit was tried for different time intervals. All of them start at the same epoch and extend to a total time between 6 and 36 hours. Obviously, since the position is known every 9 minutes, the longer the time interval the higher the number of measurements. This means that the accuracy of the model fit is expected to decrease when trying to fit a longer interval.

The tables below show the rms of the differences between the positions obtained during the OD and the one provided by the analytical models described earlier (GPS, Extended GPS and SPOT). RSS stands for Root Sum Square, or square root of the sum of squares of the position errors in the X, Y and Z components.

<b>Time (h)</b>	<b>X (km)</b>	<b>Y (km)</b>	<b>Z (km)</b>	<b>RSS (km)</b>
<b>6</b>	0.074	0.108	0.164	0.210
<b>12</b>	0.304	0.259	0.397	0.563
<b>18</b>	0.313	0.314	0.419	0.610
<b>24</b>	0.314	0.293	0.385	0.577
<b>30</b>	0.305	0.314	0.395	0.590
<b>36</b>	0.313	0.311	0.415	0.605

*Table 4-2: GPS Model Position Error (rms)*



Time (h)	X (km)	Y (km)	Z (km)	RSS (km)
6	0.060	0.079	0.096	0.138
12	0.125	0.166	0.234	0.312
18	0.263	0.294	0.364	0.537
24	0.303	0.292	0.377	0.565
30	0.302	0.316	0.394	0.588
36	0.311	0.305	0.407	0.596

*Table 4-3: Extended GPS Model Position Error (rms)*

Time (h)	X (km)	Y (km)	Z (km)	RSS (km)
6	0.042	0.054	0.058	0.090
12	0.079	0.062	0.140	0.172
18	0.106	0.109	0.132	0.202
24	0.138	0.126	0.146	0.237
30	0.152	0.130	0.159	0.256
36	0.165	0.120	0.173	0.267

*Table 4-4: SPOT Model Position Error (rms)*

The figure on the next page shows the evolution of the RSS in the three models for different time intervals. As expected, the accuracy of the model decreases as a longer interval is taken. For the basic and extended GPS models a fast decrease in the accuracy is obtained when the time interval is extended from 6 to 18 hours. After that the position error increases very slowly.

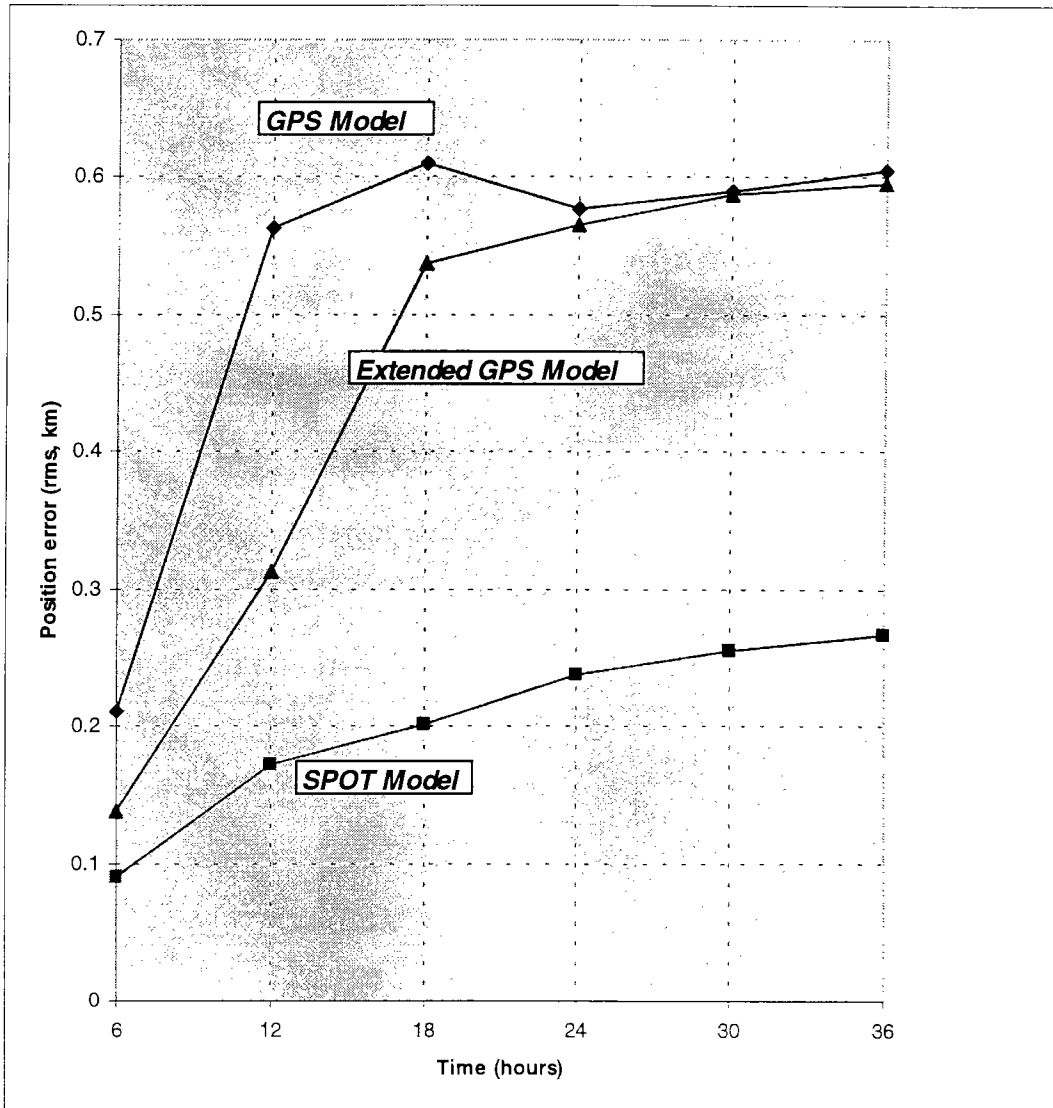


Figure 4-2: RSS of the Position Error for several analytical models

The SPOT model provides a much higher accuracy than the GPS models. This confirms our expectations. It is intended to be used with LEO satellites as opposed to the GPS models, which work better with MEO satellites. The more accurate modelling of the perturbations acting on METOP produces a better approximation to the orbit obtained by OD. The position error increases slowly as the fit interval is increased. The root sum square of the position error during the 36-hour fit interval is 267 metres. The addition of the OD error and the errors caused by on-board clock inaccuracies would increase this error very slightly.

The results clearly conclude that **the SPOT model should be used for METOP**. The table below shows the final accuracy that can be obtained in a 36-hour period with a single set of 13 parameters. The errors are projected in along-track, cross-track and radial components.

	<b>Radial (km)</b>	<b>Along-track (km)</b>	<b>Cross-track (km)</b>
<b>rms</b>	.099	.238	.074
<b>Max</b>	.297	.559	.211

*Table 4-5: SPOT Model Position Errors for a 36-hour period*

An easy way to improve the accuracy of the fitted orbit would be using several sets of parameters to approximate it. For example, instead of one set of parameters for 36 hours, two different sets could be applied, one for the first 18 hours and the other one for the last 18 hours. For the SPOT model this would take the RSS of the position error down to about 200 metres. If 6 sets of parameters were used (6-hour intervals) the RSS of the position error would go below 100 metres. A balance between accuracy and number of parameters to be uplinked will lead to the best solution. The table below shows the results corresponding to a 6-hour period. They give an idea of the accuracy that would be obtained if 6 sets of parameters instead of 1 were used for the 36-hour period.

	<b>Radial (km)</b>	<b>Along-track (km)</b>	<b>Cross-track (km)</b>
<b>rms</b>	.045	.068	.038
<b>Max</b>	.109	.137	.053

*Table 4-6: SPOT Model Position Errors for a 6-hour period*

A compromise solution between the two options shown would be two sets of parameters, one for the first 18 hours and the other one for the last 18 hours. This would provide the accuracy shown in the table below:

	<b>Radial (km)</b>	<b>Along-track (km)</b>	<b>Cross-track (km)</b>
<b>Rms</b>	.090	.168	.066
<b>Max</b>	.209	.363	.147

*Table 4-7: SPOT Model Position Errors for an 18-hour period*

The computation of the parameters requires an amount of time that grows with the complexity of the model used and the number of orbit data provided. For the SPOT model, the 36-hour fit took about 30 minutes in order to achieve convergence when running in a Sun Ultra-Sparcstation 20. The 6-hour fit took less than a minute. Obviously, the CPU time grows very fast as the number of orbit data considered is increased. These times could be cut significantly once an operational set-up is done and the initial values of the parameters can be estimated more accurately, which would reduce the number of iterations.

The table below shows, as a reference, the values of the parameters obtained for the 36-hour case with the SPOT model. Note that these values will be different for a different epoch.

Parameter	Value	Parameter	Value
P1	7.188990e03 km	P8	+1.034581e-03 rad/sec
P2	-4.638379e-04	P9	+8.501167e-04
P3	+1.179349e-03	P10	-1.039689e-04 rad
P4	+1.723015e+00 rad	P11	+5.430823e+01 rad
P5	+2.503810e+00 rad	P12	-1.525059e-15 rad/sec <sup>2</sup>
P6	+4.755502e-03 rad	P13	-1.057372e-07 1/sec
P7	+2.002655e-07 rad/sec		

Table 4-8: SPOT Model Parameters calculated for the 36-hour case

#### 4.4 KLINKRAD'S METHOD

As an extension to the previous sections, Klinkrad's method has been analysed for completion.

This method was used during the ERS-1 mission which had very strict requirements regarding the accuracy of the orbit determination and prediction, due to the nature of the mission. The method is fully described in reference [DS. 22].

The basic method uses a 1-st order state prediction including  $J_2$ ,  $J_2^2$ ,  $J_3$  and  $J_4$ . An extension has also been defined in order to improve the accuracy by including an estimation of the effects of 2<sup>nd</sup> order gravitational terms, solar attraction and air-drag. Semi-empirical functions tuned for the actual satellite are used in this extended method.

The process for orbit determination at the local users takes place after ESOC performs an Orbit Determination for a 3-day moving window and provides the predicted Cartesian state vectors for the epochs of true ascending node crossings during the next 16 orbits (1 day). This is done every 24 hours. Klinkrad's method predicts the osculating orbit parameters for a given orbit given the orbital elements at the ascending node. The method is intended to be used for 1-orbit arc lengths only, which means that the state vector (plus the epoch) at the ascending node has to be provided for each orbit. It is very good at predicting short-periodic variations but is not intended for long-term predictions.

Reference [DS. 22] states that the following accuracy was achieved with this method for ERS-1 (similar results would be expected for METOP):

		<b>Radial (km)</b>	<b>Along- track (km)</b>	<b>Cross- track (km)</b>
<b>Basic</b>	<b>Rms</b>	.098	.523	.080
<b>Basic</b>	<b>Max</b>	.336	1.795	.221
<b>Extended</b>	<b>Rms</b>	.009	.027	.009
<b>Extended</b>	<b>Max</b>	.028	.081	.040

*Table 4-9: Klinkrad's basic and extended Model Position Errors for one orbit (ERS-1)*

Compared to the results obtained with SPOT model for METOP, the basic method provides worse results for any prediction length. The extended method results in a level of accuracy that is better than the one obtained for 6 hours, halving the along-track error. However, only 13 parameters (plus the epoch) were needed for the SPOT model, whereas more than 21 parameters (3.6 orbits, 7 parameters per orbit) would be needed for Klinkrad's method.

Although no complete analysis has been performed on the accuracy that Klinkrad's method would provide for METOP, a few conclusions can be drawn from the previous paragraphs:

- **Klinkrad's extended method provides a good short-term (one orbit) approximation.** The accuracy of the along-track component is around twice the one provided by SPOT Model, although the number of parameters needed is about twice for a 6-hour period.
- **If the propagation interval is more than 6 hours, Klinkrad's method would require an excessive number of parameters:** more than 70 parameters for 18 hours, more than 140 parameters for 36 hours. A variation of the method involving the propagation for more than one orbit with the same state vector is expected to increase significantly the error. This is due to the fact that the method is focused on short-period variations of the orbital parameters.
- **SPOT Model provides an accuracy that is considered satisfactory for METOP requirements** with a single set of 13 parameters (plus the epoch). It is flexible and can be adapted to any propagation interval. Klinkrad's method will only be advisable if very strict requirements are set on the accuracy that the local users need and no constraints are present regarding the number of parameters to be sent to them.

## 4.5 END-TO-END PERFORMANCES

All the steps of the orbit determination/prediction and local user determination have already been analysed thoroughly. In this section they will be combined in order to estimate what the end-to-end performances will be. Two possible situations can be found:

1. **The local user wants to determine the position of the satellite for a time in the past.** In this case, the process will include the on-ground orbit determination, the on-ground estimation of the analytical model and the calculation of the position by the local user using the analytical model and the parameters. The total position error will be:

$$\text{TOTAL POSITION ERROR} = [ ( \text{OD}_{\text{error}} )^2 + ( \text{Analytical Model Error} )^2 ]^{1/2}$$

2. **The local user wants to determine the position of the satellite for a time in the future.** In this case, the process will include the on-ground orbit determination and prediction, the on-ground estimation of the analytical model and the calculation of the position by the local user using the analytical model and the parameters. The total position error will be:

$$\text{TOTAL POSITION ERROR} = [ ( \text{OP}_{\text{error}} )^2 + ( \text{Analytical Model Error} )^2 ]^{1/2}$$

In the first case, the OD error (in the order of a few metres) is negligible compared to the Analytical Model Error (in the order of a few hundred metres). In the second case, the two errors will be comparable unless the propagation time is short and the solar activity is low, which would make the prediction error negligible. The final error can be easily extrapolated from the studies shown, once a final decision on the determination strategy is made.

An additional error source, related to the mismatch between the local user's clock and the clock used during the OD/OP and the estimation of the analytical model parameters, could be considered. A difference of 15 ms, which could be expected, would produce a position error of around 112.5 metres (considering the lineal velocity is around 7.5 km/sec) which would have to be added to the other error sources.



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## 5. CONCLUSIONS

These are the conclusions of this technical note:

- A study has been completed on the main factors that affect the **Orbit Prediction accuracy**. Some recommendations have been presented regarding the model that should be used for the OD/OP process. Real ERS-2 data have been shown and should be a good reference for METOP. A simulation of the OD/OP process has been done for METOP and results have been presented for several error levels in the  $C_D$ , which is considered the main driver of the OP error. It has been shown that a high variation of the OP accuracy should be expected due to the evolution of the solar activity.
- Several **Analytical Models for Local Users** have been presented. Their accuracy has been tested for METOP. The SPOT model was found to be more accurate than the GPS and extended GPS models, due to its better adaptation to LEO satellites. The accuracy of the fit depends heavily on the length of time considered. Therefore, the error can be reduced by dividing the fit interval in several subintervals and performing a fit with several sets of parameters, one for each subinterval. The error levels for periods of 6, 18 and 36 hours have been shown and should help decide the number of parameters that will be used to approximate the orbit. Finally, the end-to-end performances of the OD/OP process have been analysed.